Exercise 26

Show that every point on the line $\mathbf{v} = (1, -1, 2) + t(2, 3, 1)$ satisfies the equation $5x - 3y - z - 6 = 0$.

Solution

The parameterization for the line can be written as

$$\begin{aligned}\mathbf{v} &= (1, -1, 2) + t(2, 3, 1) \\ &= (1, -1, 2) + (2t, 3t, t) \\ &= (1 + 2t, -1 + 3t, 2 + t).\end{aligned}$$

The x-, y-, and z-components of the line are

$$x = 1 + 2t \quad \text{and} \quad y = -1 + 3t \quad \text{and} \quad z = 2 + t,$$

respectively. Substitute these into the equation for the plane.

$$\begin{aligned}5x - 3y - z - 6 &= 5(1 + 2t) - 3(-1 + 3t) - (2 + t) - 6 \\ &= 5 + 10t + 3 - 9t - 2 - t - 6 \\ &= 0\end{aligned}$$

Since the right side is 0 for all values of t, every point (x, y, z) on the line lies in the plane.